Exercise 48

If $f(2) = 10$ and $f'(x) = x^2 f(x)$ for all x, find $f''(2)$.

Solution

Differentiate the first derivative using the product rule.

$$\begin{aligned} f''(x) &= \frac{d}{dx}[f'(x)] \\ &= \frac{d}{dx}[x^2 f(x)] \\ &= \left[\frac{d}{dx}(x^2)\right] f(x) + x^2 f'(x) \\ &= (2x)f(x) + x^2 f'(x) \end{aligned}$$

Then evaluate it at $x = 2$.

$$\begin{aligned} f''(2) &= (4)f(2) + (2)^2 f'(2) \\ &= 4(10) + 4[(2)^2 f(2)] \\ &= 40 + 4[4(10)] \\ &= 200 \end{aligned}$$